Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topics: Straight line, P.O.S, Circle, Parabola, Ellipse & Hyperbola

***Education is not just about going to school and getting a degree. It's about widening your knowledge and absorbing the truth about life.*

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Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. If the area of triangle formed by the points $(2a, b)$ $(a+b, 2b+a)$ and $(2b, 2a)$ be λ, then the area of the triangle whose vertices are $(a+b, a-b)$, $(3b-a, b+3a)$ and $(3a-b, 3b-a)$ will be

(a) $\frac{3}{2}\lambda$ (b) 3λ

(c) 4λ (d) none of these

2. The vertices of a triangle are $A\ (x_1, x_1 \tan\alpha)$, $B(x_2, x_2 \tan\beta)$ and $C\ (x_3, x_3 \tan\gamma)$. If the circumcentre of ΔABC coincides with the origin and $H\ (a, b)$ be its orthocentre, then $\frac{a}{b}$ is equal to

(a) $\frac{\cos\alpha + \cos\beta + \cos\gamma}{\cos\alpha\ \cos\beta\ \cos\gamma}$ (b) $\frac{\sin\alpha + \sin\beta + \sin\gamma}{\sin\alpha\ \sin\beta\ \sin\gamma}$

(c) $\frac{\tan\alpha + \tan\beta + \tan\gamma}{\tan\alpha\ \tan\beta\ \tan\gamma}$ (d) $\frac{\cos\alpha + \cos\beta + \cos\gamma}{\sin\alpha + \sin\beta + \sin\gamma}$

3. The image of $P\ (a, b)$ on $y = -x$ is Q and the image of Q on the line $y = x$ is R. Then the mid point of R is

(a) $(a+b, b+a)$ (b) $\left(\frac{a+b}{2}, \frac{b+a}{2}\right)$

(c) $(a-b, b-a)$ (d) $(0,0)$

4. A straight line L with negative slope passes through the point $(8, 2)$ and cuts the positive coordinate axes at points P and Q. As L varies, the absolute minimum value of $OP + OQ$ is (O is origin)

(a) 10 (b) 18

(c) 16 (d) 12

5. Drawn from origin are two mutually perpendicular lines forming an isosceles triangle together with the straight line $2x + y = a$, then the area of this triangle is

(a) $\frac{a^2}{2}$ sq unit (b) $\frac{a^2}{3}$ sq unit

(c) $\frac{a^2}{5}$ sq unit (d) none of these

6. If the distance of any point (x, y) from origin is defined as $d\,(x, y) = |\,x\,| + |\,y\,|$, then the locus $d\,(x, y) = 1$ is a

(a) circle of area π sq unit

(b) square of area 1 sq unit

(c) square of area 2 sq unit

(d) none of the above

7. The line $3x - 4y + 7 = 0$ is rotated through an angle $\frac{\pi}{4}$ in the clockwise direction about the point $(-1, 1)$. The equation of the line in its new position is

(a) $7y + x - 6 = 0$ (b) $7y - x - 6 = 0$

(c) $7y + x + 6 = 0$ (d) $7y - x + 6 = 0$

8. The number of integral values of m for which the x coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer is

(a) 2 (b) 0

(c) 4 (d) 1

9. The distance between the circumcentre and orthocentre of the triangle whose vertices are $(0, 0)$, $(6, 8)$ and $(-4, 3)$ is

(a) $\frac{125}{8}$ unit (b) $\frac{\sqrt{5}}{2}$ unit

(c) $\frac{5\sqrt{5}}{2}$ unit (d) $5\sqrt{5}$ unit

10. For all real values of a and b lines $(2a + b)x + (a + 3b)\,y + (b - 3a) = 0$ and $m\,x + 2y + 6 = 0$ are concurrent, then m is equal to

(a) -2 (b) -3

(c) -4 (d) -5

11. A system of lines is given as $y = m_i\,x + c_i$, where m_i can take any value out of 0, 1,–1 and when m_i is positive, then c_i can be 1 or –1 when m_i equal 0,c_i can be 0 or 1 and when m_i equals to – 1, c_i can take 0 or 2. Then the area enclosed by all these straight lines is

(a) $\frac{3}{\sqrt{2}}(\sqrt{2} - 1)$ sq unit (b) $\frac{3}{\sqrt{2}}$ sq unit

(c) $\frac{3}{2}$ sq unit (d) none of these

12. If all the 3 vertices of an isosceles right angle triangle be integral points and length of base is also an integer, then which of the point is never a rational point (A point $P(x, y)$ is integer point if both x and y are integers and point is rational, if both x and y are rational)
(a) Centroid (b) Incentre
(c) Circumcentre (d) Orthocentre

13. A man starts from the point $P(-3, 4)$ and reaches point $Q(0, 1)$ touching x axis at R such that $PR + RQ$ is minimum, then the point R is
(a) $\left(\frac{3}{5}, 0\right)$ (b) $\left(-\frac{3}{5}, 0\right)$
(c) $\left(-\frac{2}{5}, 0\right)$ (d) $(-2, 0)$

14. A beam of light is sent along the line $x - y = 1$, which after refracting from the x-axis enters the opposite side by turning through 30° towards the normal at the point of incidence on the x-axis. Then the equation of the refracted ray is
(a) $(2-\sqrt{3})x - y = 2+\sqrt{3}$ (b) $(2+\sqrt{3})x - y = 2+\sqrt{3}$
(c) $(2-\sqrt{3})x + y = (2+\sqrt{3})$ (d) none of these

15. Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals
(a) $\frac{|m+n|}{(m-n)^2}$ (b) $\frac{2}{|m+n|}$
(c) $\frac{1}{|m+n|}$ (d) $\frac{1}{|m-n|}$

16. Given a family of lines $a(2x + y + 4) + b(x - 2y - 3) = 0$, the number of lines belonging to the family at a distance $\sqrt{10}$ from $P(2, -3)$ is
(a) 0 (b) 1
(c) 2 (d) 4

17. The point (4, 1) undergoes the following three successive transformations
(i) Reflection about the line $y = x - 1$
(ii) Translation through a distance 1 units along the positive direction of x - axis
(iii) Rotation through an angle $\frac{\pi}{4}$ about the origin in the anticlockwise direction.
Then, the coordinates of the final point are
(a) (4, 3) (b) $\left(\frac{7}{2}, \frac{7}{2}\right)$
(c) $(0, 3\sqrt{2})$ (d) (3, 4)

18. If $5a + 4b + 20c = t$, then the value of t for which the line $ax + by + c - 1 = 0$ always passes through a fixed point is
(a) 0 (b) 20
(c) 30 (d) none of these

19. Consider the family of lines $(x + y - 1) + \lambda(2x + 3y - 5) = 0$ and $(3x + 2y - 4) + \mu(x + 2y - 6) = 0$, equation of a straight line that belongs to both the families is
(a) $x - 2y - 8 = 0$ (b) $x - 2y + 8 = 0$
(c) $2x + y - 8 = 0$ (d) $2x - y - 8 = 0$

20. If the distance of any point (x, y) from the origin is defined as $d(x, y) = \max\{|x|, |y|\}$, $d(x, y) = a$, non zero constant, then the locus is
(a) a circle (b) a straight line
(c) a square (d) a triangle

21. The point (4, 1) undergoes the following three transformations successively
(I) Reflection about the line $y = x$
(II) Translation through a distance 2 units along the positive direction of x-axis
(III) Rotation through an angle $\pi/4$ about the origin in the anticlockwise direction.
The final position of the point is given by the coordinates
(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $(-2, 7\sqrt{2})$
(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$

22. One of the bisector of the angle between the lines $a(x-1)^2 + 2h(x-1)(y-1) + b(y-2)^2 = 0$ is $x + 2y - 5 = 0$. The other bisector is
(a) $2x - y = 0$ (b) $2x + y = 0$
(c) $2x + y - 4 = 0$ (d) $x - 2y + 3 = 0$

23. Line L has intercepts a and b on the coordinate axes, when the axes are rotated through a given angle; keeping the origin fixed, the same line has intercepts p and q, then
(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$
(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

24. The point $A(2, 1)$ is translated parallel to the line $x - y = 3$ by a distance 4 units. If the new position A' is in third quadrant, then the coordinates of A' are
(a) $(2 + 2\sqrt{2}, 1 + 2\sqrt{2})$ (b) $(-2 + \sqrt{2}, -1 - 2\sqrt{2})$
(c) $(2 - 2\sqrt{2}, 1 - 2\sqrt{2})$ (d) none of these

25. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The coordinates of the point A are
(a) $\left(\frac{13}{5}, 0\right)$ (b) $\left(\frac{5}{13}, 0\right)$
(c) $(-7, 0)$ (d) none of these

26. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

27. The reflection of the point $(4, -13)$ in the line $5x + y + 6 = 0$ is
(a) $(-1, -14)$ (b) (3, 4)
(c) (1, 2) (d) $(-4, 13)$

28. All the points lying inside the triangle formed by the points (0, 4), (2, 5) and (6, 2) satisfy
(a) $3x + 2y + 8 \geq 0$ (b) $2x + y - 10 \geq 0$
(c) $2x - 3y - 11 \geq 0$ (d) $-2x + y - 3 \geq 0$

29. A line passing through $P(4, 2)$ meets the x and y-axis at A and B respectively. If O is the origin, then locus of the centre of the circumcircle of ΔOAB is
(a) $x^{-1} + y^{-1} = 2$ (b) $2x^{-1} + y^{-1} = 1$
(c) $x^{-1} + 2y^{-1} = 1$ (d) $2x^{-1} + 2y^{-1} = 1$

30. Let n be the number of points having rational coordinates equidistant from the point $(0, \sqrt{3})$, then
(a) $n \le 1$ (b) $n = 1$
(c) $n \le 2$ (d) $n > 2$

31. The coordinates of the middle points of the sides of a triangle are (4, 2), (3, 3) and (2, 2), then the coordinates of its centroid are
(a) (3, 7/3) (b) (3, 3)
(c) (4, 3) (d) none of these

32. The line segment joining the points (1, 2) and (– 2, 1) is divided by the line $3x + 4y = 7$ in the ratio
(a) 3 : 4 (b) 4 : 3
(c) 9 : 4 (d) 4 : 9

33. If a straight line passes through (x_1, y_1) and its segment between the axes is bisected at this point, then its equation is given by
(a) $\frac{x}{x_1} + \frac{y}{y_1} = 2$ (b) $2(xy_1 + yx_1) = x_1y_1$
(c) $xy_1 + yx_1 = x_1y_1$ (d) none of these

34. If p_1, p_2, p_3 be the length perpendiculars from the points $(m^2, 2m), (mm', m + m')$ and $(m'^2, 2m')$ respectively on the line
$x\cos\alpha + y\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha} = 0$, then p_1, p_2, p_3 are in
(a) AP (b) GP
(c) HP (d) none of these

35. The acute angle θ through which the coordinate axes should be rotated for the point $A(2, 4)$ to attain the new abscissa 4 is given by
(a) $\tan\theta = 3/4$ (b) $\tan\theta = 5/6$
(c) $\tan\theta = 7/8$ (d) none of these

36. If $\frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^m}{n!}$, then orthocentre of the triangle having sides $x - y + 1 = 0$, $x + y + 3 = 0$ and $2x + 5y - 2 = 0$ is
(a) $(2m - 2n, m - n)$ (b) $(2m - 2n, n - m)$
(c) $(2m - n, m + n)$ (d) $(2m - n, m - n)$

37. The equation of straight line equally inclined to the axes and equidistant from the point (1, – 2) and (3, 4) is
(a) $x + y = 1$ (b) $y - x - 1 = 0$
(c) $y - x = 2$ (d) $y - x + 1 = 0$

38. If one of the diagonal of a square is along the line $x = 2y$ and one of its vertices is (3, 0), then its sides through this vertex are given by, the equations
(a) $y - 3x + 9 = 0, 3y + x - 3 = 0$
(b) $y + 3x + 9 = 0, 3y + x - 3 = 0$
(c) $y - 3x + 9 = 0, 3y - x + 3 = 0$
(d) $y - 3x + 3 = 0, 3y + x + 9 = 0$

39. The graph of the function $y = \cos x \cos(x + 2) - \cos^2(x + 1)$ is
(a) a straight line passing through $(0, -\sin^2 1)$ with slope 2
(b) a straight line passing through (0, 0)
(c) a parabola with vertex $(1, -\sin^2 1)$
(d) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ are parallel to the x –axis

40. $P(m, n)$ (where m, n are natural numbers) is any point in the interior of the quadrilateral formed by the pair of lines $xy = 0$ and the two lines $2x + y - 2 = 0$ and $4x + 5y = 20$. The possible number of positions of the P is
(a) six (b) five
(c) four (d) eleven

41. The image of the point $A(1, 2)$ by the line mirror $y = x$ is the point B and the image of B by the line mirror $y = 0$ is the point (α, β), then
(a) $\alpha = 1, \beta = -2$ (b) $\alpha = 0, \beta = 0$
(c) $\alpha = 2, \beta = -1$ (d) none of these

42. $ABCD$ is a square whose vertices A, B, C and D are (0, 0), (2, 0), (2, 2) and (0, 2) respectively. This square is rotated in the $X - Y$ plane with an angle of 30° in anticlockwise direction about an axis passing through the vertex A the equation of the diagonal BD of this rotated square is....... If E is the centre of the square, the equation of the circecumcircle of the triangle ABE is
(a) $\sqrt{3}x + (1 - \sqrt{3})y = \sqrt{3}, x^2 + y^2 = 4$
(b) $(1 + \sqrt{3})x - (1 - \sqrt{2})y = 2, x^2 + y^2 = 9$
(c) $(2 - \sqrt{3})x + y = 2(\sqrt{3} - 1), x^2 + y^2 - x\sqrt{3} - y = 0$
(d) none of the above

43. The straight line $y = x - 2$ rotates about a point where it cuts x-axis and becomes perpendicular on the straight line $ax + by + c = 0$, then its equation is
(a) $ax + by + 2a = 0$ (b) $ay - bx + 2b = 0$
(c) $ax + by + 2b = 0$ (d) none of these

44. The line $x + y = a$ meets the axis of x and y at A and B respectively a triangle AMN is inscribed in the triangle OAB, O being the origin, with right angle at N, M and N lie respectively on OB and AB. If the area of the triangle AMN is 3/8 of the area of the triangle OAB, then AN/BN is equal to
(a) 1 (b) 2
(c) 3 (d) 4

45. If two vertices of an equilateral triangle have integral coordinates, then the third vertex will have
(a) integral coordinates
(b) coordinates which are rational
(c) at least one coordinate irrational
(d) coordinates which are irrational

46. If $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ are the values of n for which $\sum_{r=0}^{n-1} x^{2r}$ is divisible by $\sum_{r=0}^{n-1} x^r$, then the triangle having vertices (α_1, β_1), (α_2, β_2) and (α_3, β_3) cannot be
(a) an isosceles triangle
(b) a right angled isosceles triangle
(c) a right angled triangle
(d) an equilateral triangle

47. The points (α, β), (γ, δ), (α, δ) and (γ, β), where $\alpha, \beta, \gamma, \delta$ are different real numbers, are
(a) collinear
(b) vertices of a square
(c) vertices of a rhombus
(d) concyclic

48. The equations of the three sides of a triangle are $x = 2$, $y + 1 = 0$ and $x + 2y = 4$. The coordinates of the circumcentre of the triangle are
(a) $(4, 0)$
(b) $(2, -1)$
(c) $(0, 4)$
(d) none of these

49. If $P(1 + \alpha/\sqrt{2}, 2 + \alpha/\sqrt{2})$ be any point on a line, then the range of values of t for which the point P lies between the parallel lines $x + 2y = 1$ and $2x + 4y = 15$ is
(a) $-\frac{4\sqrt{2}}{3} < \alpha < \frac{5\sqrt{2}}{6}$
(b) $0 < \alpha < \frac{5\sqrt{2}}{6}$
(c) $-\frac{4\sqrt{2}}{3} < \alpha < 0$
(d) none of these

50. A circle of radius 5 unit touches both the axes and lies in the first quadrant. If the circle makes one complete roll on x-axis along the positive direction of x-axis, then its equation in the new position is
(a) $x^2 + y^2 + 20\pi x - 10y + 100\pi^2 = 0$
(b) $x^2 + y^2 + 20\pi x + 10y + 100\pi^2 = 0$
(c) $x^2 + y^2 - 20\pi x - 10y + 100\pi^2 = 0$
(d) none of the above

51. If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then
(a) $2 < r < 8$
(b) $r < 2$
(c) $r = 2$
(d) $r > 2$

52. A variable circle always touches the line $y = x$ and passes through the point (0, 0). The common chords of above circle and $x^2 + y^2 + 6x + 8y - 7 = 0$ will pass through a fixed point whose coordinates are
(a) $\left(-\frac{1}{2}, \frac{3}{2}\right)$
(b) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(d) none of these

53. The locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally is
(a) $3x + 4y - 5 = 0$
(b) $9x - 10y + 7 = 0$
(c) $9x + 10y - 7 = 0$
(d) $9x - 10y + 11 = 0$

54. If from any point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is
(a) 2α
(b) α
(c) $\alpha/2$
(d) $\alpha/4$

55. The equations of the circles which touch both the axes and the line $x = a$ are
(a) $x^2 + y^2 \pm ax \pm ay + \frac{a^2}{4} = 0$
(b) $x^2 + y^2 + ax \pm ay + \frac{a^2}{4} = 0$
(c) $x^2 + y^2 - ax \pm ay + \frac{a^2}{4} = 0$
(d) none of the above

56. A, B, C and D are the points of intersection with the coordinate axes of the lines $ax + by = ab$ and $bx + ay = ab$, then
(a) A, B, C, D are concyclic
(b) A, B, C, D form a parallelogram
(c) A, B, C, D form a rhombus
(d) none of the above

57. The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle is equal to
(a) $\pi/6$
(b) $\pi/4$
(c) $\pi/3$
(d) $\pi/2$

58. The number of common tangents that can be drawn to the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$ is
(a) 1
(b) 2
(c) 3
(d) 4

59. If the distances from the origin of the centres of three circles $x^2 + y^2 + 2\lambda_i x - c^2 = 0\ (i = 1, 2, 3)$ are in GP, then the lengths of the tangents drawn to them from any point on the circle $x^2 + y^2 = c^2$ are in
(a) AP
(b) GP
(c) HP
(d) none of these

60. If $4l^2 - 5m^2 + 6l + 1 = 0$ and the line $lx + my + 1 = 0$ touches a fixed circle, then
(a) the centre of the circle is at the point (4, 0)
(b) the radius of the circle is equal to $\sqrt{5}$
(c) the circle passes through origin
(d) none of the above

61. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle drawn on this chord as diameter is
(a) $x^2 + y^2 + ax = 0$
(b) $x^2 + y^2 + ay = 0$
(c) $x^2 + y^2 - ax = 0$
(d) $x^2 + y^2 - ay = 0$

62. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = \lambda^2$ orthogonally, equation of the locus of its centre is
(a) $2ax + 2by = a^2 + b^2 + \lambda^2$
(b) $ax + by = a^2 + b^2 + \lambda^2$
(c) $x^2 + y^2 + 2ax + 2by + \lambda^2 = 0$
(d) $x^2 + y^2 - 2ax - 2by + a^2 + b^2 - \lambda^2 = 0$

63. If O is the origin and OP, OQ are distinct tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the circumcentre of the triangle OPQ is
(a) $(-g, -f)$
(b) (g, f)
(c) $(-f, -g)$
(d) none of these

64. The circle passing through the distinct points $(1, t)$, $(t, 1)$ and (t, t) for all values of t, passes through the point
(a) (1, 1)
(b) (−1, −1)
(c) (1, −1)
(d) (−1, 1)

65. Equation of a circle through the origin and belonging to the coaxial system, of which the limiting points are (1, 2), (4, 3) is
(a) $x^2 + y^2 - 2x + 4y = 0$
(b) $x^2 + y^2 - 8x - 6y = 0$
(c) $2x^2 + 2y^2 - x - 7y = 0$
(d) $x^2 + y^2 - 6x - 10y = 0$

66. Equation of the normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is
(a) $3x + 2y - 5 = 0$
(b) $3x + y - 4 = 0$
(c) $3x + 2y - 2 = 0$
(d) $3x - y - 8 = 0$

67. α, β and γ are parametric angles of three points P, Q and R respectively, on the circle $x^2 + y^2 = 1$ and A is the point $(-1, 0)$. If the lengths of the chords AP, AQ and AR are in GP, then $\cos\alpha/2$, $\cos\beta/2$ and $\cos\gamma/2$ are in

(a) AP (b) GP
(c) HP (d) none of these

68. The area bounded by the circles $x^2 + y^2 = r^2, r = 1, 2$ and the rays given by $2x^2 - 3xy - 2y^2 = 0, y > 0$ is

(a) $\frac{\pi}{4}$ sq unit (b) $\frac{\pi}{2}$ sq unit
(c) $\frac{3\pi}{4}$ sq unit (d) π sq unit

69. The equation of the circle touching the lines $|y| = x$ at a distance $\sqrt{2}$ unit from the origin is

(a) $x^2 + y^2 - 4x + 2 = 0$ (b) $x^2 + y^2 + 4x - 2 = 0$
(c) $x^2 + y^2 + 4x + 2 = 0$ (d) none of these

70. The values of λ for which the circle $x^2 + y^2 + 6x + 5 + \lambda(x^2 + y^2 - 8x + 7) = 0$ dwindles into a point are

(a) $1 \pm \frac{\sqrt{2}}{3}$ (b) $2 \pm \frac{2\sqrt{2}}{3}$
(c) $2 \pm \frac{4\sqrt{2}}{3}$ (d) $1 \pm \frac{4\sqrt{2}}{3}$

71. The equation of the circle passing through $(2, 0)$ and $(0, 4)$ and having the minimum radius is

(a) $x^2 + y^2 = 20$
(b) $x^2 + y^2 - 2x - 4y = 0$
(c) $(x^2 + y^2 - 4) + \lambda(x^2 + y^2 - 16) = 0$
(d) none of the above

72. The shortest distance from the point $(2, -7)$ to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is

(a) 1 (b) 2
(c) 3 (d) 4

73. The circle $x^2 + y^2 = 4$ cuts the line joining the points $A(1, 0)$ and $B(3, 4)$ in two points P and Q. Let $\frac{BP}{PA} = \alpha$ and $\frac{BQ}{QA} = \beta$, then α and β are roots of the quadratic equation

(a) $x^2 + 2x + 7 = 0$ (b) $3x^2 + 2x - 21 = 0$
(c) $2x^2 + 3x - 27 = 0$ (d) none of these

74. The equation of the image of the circle $(x-3)^2 + (y-2)^2 = 1$ by the mirror $x + y = 19$ is

(a) $(x-14)^2 + (y-13)^2 = 1$
(b) $(x-15)^2 + (y-14)^2 = 1$
(c) $(x-16)^2 + (y-15)^2 = 1$
(d) $(x-17)^2 + (y-16)^2 = 1$

75. If P and Q are two points on the circle $x^2 + y^2 - 4x - 4y - 1 = 0$ which are farthest and nearest respectively from the point $(6, 5)$, then

(a) $P \equiv \left(-\frac{22}{5}, 3\right)$ (b) $Q \equiv \left(\frac{22}{5}, \frac{19}{5}\right)$
(c) $P \equiv \left(\frac{14}{3}, -\frac{11}{5}\right)$ (d) $Q \equiv \left(-\frac{14}{3}, -4\right)$

76. If α, β are the roots of $ax^2 + bx + c = 0$ and α', β' those of $a'x^2 + b'x + c' = 0$, the equation of the circle having $A(\alpha, \alpha')$ and $B(\beta, \beta')$ as diameter is

(a) $cc'(x^2 + y^2) + ac'x + a'cy + a'b + ab' = 0$
(b) $cc'(x^2 + y^2) + a'cx + ac'y + a'b + ab' = 0$
(c) $bb'(x^2 + y^2) + a'bx + ab'y + a'c + ac' = 0$
(d) $aa'(x^2 + y^2) + a'bx + ab'y + a'c + ac' = 0$

77. The circles $(x-a)^2 + (y-b)^2 = c^2$ and $(x-b)^2 + (y-a)^2 = c^2$ touch each other, then

(a) $a = b \pm 2c$ (b) $a = b \pm \sqrt{2}c$
(c) $a = b \pm c$ (d) none of these

78. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is

(a) $x^2 + y^2 - 16x - 18y - 4 = 0$
(b) $x^2 + y^2 - 7x + 11y + 6 = 0$
(c) $x^2 + y^2 + 2x - 8y + 9 = 0$
(d) none of the above

79. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = r^2$ at A and B, then $PA \cdot PB$ is equal to

(a) $(\alpha + \beta)^2 - r^2$ (b) $\alpha^2 + \beta^2 + r^2$
(c) $(\alpha - \beta)^2 + r^2$ (d) none of these

80. A line meets the coordinate axes in A and B. A circle is circumscribed about the triangle OAB. If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is

(a) $m(m+n)$ (b) $(m+n)$
(c) $n(m+n)$ (d) $\frac{1}{2}(m+n)$

81. The locus of the point of intersection of the lines $x = a\left(\frac{1-t^2}{1+t^2}\right)$ and $y = \frac{2at}{1+t^2}$ represents (t being a parameter)

(a) circle (b) parabola
(c) ellipse (d) hyperbola

82. If $(2, 1)$ is a limiting point of a coaxial system of circles containing $x^2 + y^2 - 6x - 4y - 3 = 0$, then the other limiting point is

(a) $(2, 4)$ (b) $(-5, -6)$
(c) $(3, 5)$ (d) $(-2, 4)$

83. Lenght of the tangent drawn from any point of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + d = 0, (d > c)$ is

(a) $\sqrt{(c-d)}$ (b) $\sqrt{(d-c)}$
(c) $\sqrt{(g-f)}$ (d) $\sqrt{(f-g)}$

Objective Questions Type II [One or more than one correct answer(s)]

In each of the questions below four choices of which one or more than one are correct. You have to select the correct answer(s) accordingly.

1. If $x^2 + \alpha y^2 + 2\beta y = a^2$ represents a pair of perpendicular straight lines, then
   (a) $\alpha = 1, \beta = a$ (b) $\alpha = 1, \beta = -a$
   (c) $\alpha = -1, \beta = -a$ (d) $\alpha = -1, \beta = a$

2. Type of quadrilateral formed by the two pairs of lines $6x^2 - 5xy - 6y^2 = 0$
   and $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ is
   (a) square (b) rhombus
   (c) parallelogram (d) rectangle

3. If the line $y = mx$ is one of the bisector of the lines $x^2 + 4xy - y^2 = 0$, then the value of m is equal to
   (a) $\frac{-1+\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{2}$
   (c) $\frac{-1-\sqrt{5}}{2}$ (d) $\frac{1-\sqrt{5}}{2}$

4. Two pairs of straight lines have the equations $y^2 + xy - 12x^2 = 0$ and $ax^2 + 2hxy + by^2 = 0$. One line will be common among them, if
   (a) $a = -3(2h + 3b)$ (b) $a = 8(h - 2b)$
   (c) $a = 2(b + h)$ (d) $a = -3(b + h)$

5. The combined equation of three sides of a triangle is $(x^2 - y^2)(2x + 3y - 6) = 0$. If $(-2, a)$ is an interior point and $(b, 1)$ is an exterior point of the triangle, then
   (a) $2 < a < \frac{10}{3}$ (b) $-2 < a < \frac{10}{3}$
   (c) $-1 < b < \frac{9}{2}$ (d) $-1 < b < 1$

6. If the two lines represented by $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make angles α, β with the x-axis, then
   (a) $\tan\alpha + \tan\beta = 4\operatorname{cosec} 2\theta$
   (b) $\tan\alpha\tan\beta = \sec^2\theta + \tan^2\theta$
   (c) $\tan\alpha - \tan\beta = 2$
   (d) $\frac{\tan\alpha}{\tan\beta} = \frac{2+\sin 2\theta}{2-\sin 2\theta}$

7. The equation $ax^2 + by^2 + cx + cy = 0$ represents a pair of straight lines, if
   (a) $a + b = 0$ (b) $c = 0$
   (c) $a + c = 0$ (d) $c(a + b) = 0$

8. If the angle between the lines $x^2 - xy + ay^2 = 0$ is $45°$, then value(s) of a is/are
   (a) -6 (b) 0
   (c) 6 (d) 12

9. Equation of pair of lines passing through $(1, -1)$ and parallel to the lines $2x^2 + 5xy + 3y^2 = 0$ is
   (a) $2(x-1)^2 + 5(x-1)(y+1) + 3(y+1)^2 = 0$
   (b) $3(x-1)^2 - 5(x-1)(y+1) + 2(y+1)^2 = 0$
   (c) $2x^2 + 5xy + 3y^2 + x + y = 0$
   (d) $3x^2 - 5xy + 2y^2 - 11x + 9y + 10 = 0$

10. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on y-axis, then
    (a) $f^2 = bc$ (b) $abc = 2fgh$
    (c) $bg^2 \neq ch^2$ (d) $2fgh = bg^2 + ch^2$

Linked-Comprehension Type

In these questions, a passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

If the lines represented by $2x^2 - 5xy + 2y^2 = 0$ be the two sides of a parallelogram and the line $5x + 2y = 1$ be one of its diagonal.

On the basis of above information, answer the following questions :

1. The equation of the other diagonal is
   (a) $10x - 11y = 0$ (b) $11x - 10y = 0$
   (c) $3x - 2y = 0$ (d) $2x - 3y = 0$

2. The centroid of the parallelogram is
   (a) $\left(\frac{5}{72}, \frac{11}{36}\right)$ (b) $\left(\frac{11}{72}, \frac{5}{36}\right)$
   (c) $\left(\frac{5}{36}, \frac{11}{72}\right)$ (d) $\left(\frac{11}{36}, \frac{5}{72}\right)$

3. The area of the parallelogram is
   (a) $\frac{1}{36}$ sq unit (b) $\frac{1}{18}$ sq unit
   (c) $\frac{1}{9}$ sq unit (d) none of these

4. The ratio of the longer side to smaller side is
   (a) 6 : 5 (b) 7 : 6
   (c) 5 : 4 (d) 4 : 3

5. The ratio of the longer diagonal to smaller diagonal is
   (a) 6 : 5 (b) 9 : 2
   (c) 13 : 5 (d) none of these

PASSAGE 2

Let $f_1(x, y) \equiv ax^2 + 2hxy + by^2 = 0$ and let $f_{i+1}(x, y) = 0$ denotes the equation of the bisectors of $f_i(x, y) = 0$ for all $i = 1, 2, 3, \ldots$

On the basis of above information, answer the following questions :

1. Equation $f_2(x, y) = 0$ is
   (a) $hx^2 - (a-b)xy + hy^2 = 0$
   (b) $hx^2 - (a-b)xy - hy^2 = 0$
   (c) $hx^2 + (a-b)xy + hy^2 = 0$
   (d) $hx^2 + (a-b)xy - hy^2 = 0$
2. Equation $f_3(x, y) = 0$ is
   (a) $(a-b)x^2 - 4hxy + (a-b)y^2 = 0$
   (b) $(a-b)x^2 - 4hxy - (a-b)y^2 = 0$
   (c) $(a-b)x^2 + 4hxy - (b-a)y^2 = 0$
   (d) $(a-b)x^2 + 4hxy - (a-b)y^2 = 0$
3. If $f_{i+1}(x, y) = 0$ represents the equation of a pair of perpendicular lines, then $f_2(x, y) = 0$ is
   (a) $bx^2 - 2hxy + ay^2 = 0$
   (b) $a x^2 + 2hxy + by^2 = 0$
   (c) $ax^2 - 2hxy + by^2 = 0$
   (d) none of the above
4. If $f_{i+1}(x, y) = 0$ represents the equation of pair of perpendicular lines, then $f_3(x, y) = 0$ is
   (a) $bx^2 - 2hxy - ay^2 = 0$
   (b) $ax^2 + 2hxy + by^2 = 0$
   (c) $ax^2 - 2hxy + by^2 = 0$
   (d) $bx^2 - 2hxy + ay^2 = 0$
5. If $f_{i+1}(x, y) = 0$ represents the equation of pair of perpendicular lines, then $f_{n+2}(x, y) = 0 \; \forall n \geq 2$ is same as
   (a) $f_{n+2}(x, y) = 0$
   (b) $f_{n+1}(x, y) = 0$
   (c) $f_n(x, y) = 0$
   (d) none of the above

PASSAGE 3

If the normals at $(x_i, y_i), i = 1, 2, 3, 4$ on the rectangular hyperbola $xy = c^2$, meet at the point (α, β).

On the basis of above information, answer the following questions :

1. The value of $\sum x_i$ is
   (a) $c\beta$ (b) $c\alpha$
   (c) α (d) β
2. The value of $\sum y_i$ is
   (a) $c\beta$ (b) $c\alpha$
   (c) α (d) β
3. The value of $\sum x_i^2$ is
   (a) c^2 (b) α^2
   (c) $-c^2$ (d) $-\beta^2$
4. The value of $\sum y_i^2$ is
   (a) β^2 (b) α^2
   (c) $-c^2$ (d) c^2
5. The value of $\prod x_i = \prod y_i =$ is
   (a) $-c$ (b) $-c^2$
   (c) $-c^3$ (d) $-c^4$

PASSAGE 1

The difference between the second degree curve and pair of asymptotes is constant.

If second degree curve represented by a hyperbola $S = 0$, then the equation of its asymptotes is $S + \lambda = 0$ where λ is constant. which will be a pair of straight lines, then we get λ. Then equation of asymptotes is $A \equiv S + \lambda = 0$ and if equation of conjugate hyperbola of S represented by S_1, then A is the arithmetic mean of S and S_1.

On the basis of above information, answer the following questions :

1. Pair of asymptotes of the hyperbola $xy - 3y - 2x = 0$ is
   (a) $xy - 3y - 2x + 2 = 0$ (b) $xy - 3y - 2x + 4 = 0$
   (c) $xy - 3y - 2x + 6 = 0$ (d) $xy - 3y - 2x + 12 = 0$
2. The asymptotes of a hyperbola having centre at the point (1, 2) are parallel to the lines $2x + 3y = 0$ and $3x + 2y = 0$. If the hyperbola passes through the point (5, 3), then its equation is
   (a) $(2x + 3y - 3)(3x + 2y - 5) = 256$
   (b) $(2x + 3y - 7)(3x + 2y - 8) = 156$
   (c) $(2x + 3y - 5)(3x + 2y - 3) = 252$
   (d) $(2x + 3y - 8)(3x + 2y - 7) = 154$
3. If angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\pi / 3$ then the eccentricity of conjugate hyperbola is
   (a) $\sqrt{2}$ (b) 2
   (c) $2/\sqrt{3}$ (d) $4/\sqrt{3}$
4. A hyperbola passing through origin has $3x - 4y - 1 = 0$ and $4x - 3y - 6 = 0$ as its asymptotes. Then the equation of its transverse and conjugate axes are
   (a) $x - y - 5 = 0$ and $x + y + 1 = 0$
   (b) $x - y = 0$ and $x + y + 5 = 0$
   (c) $x + y - 5 = 0$ and $x - y - 1 = 0$
   (d) $x + y - 1 = 0$ and $x - y - 5 = 0$
5. The tangent at any point of a hyperbola $16x^2 - 25y^2 = 400$ cuts off a triangle from the asymptotes and that the portion of it intercepted between the asymptotes, then the area of this triangle is
   (a) 10 sq unit (b) 20 sq unit
   (c) 30 sq unit (d) 40 sq unit

Numerical Grid-Based Problems

Solve the following problems and mark your response against their respective grids. Write your answer in the top row of the grid and darken the concerned numbers in the respective columns.

For example. If answer of a question is 0247, then

1. The equation of the ellipse referred to its centre whose minor axis is equal to the distance between the foci and whose latusrectum is 10 is $mx^2 + ny^2 = 100$, when $m, n \in N$, then the value of $10^m + 20^n$ must be

2. If the normal at an end of a latusrectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through one extremity of the minor axis. If e be the eccentricity of the ellipse then the value of $625(2e^2 + 1)^2$ must be

3. A ray emanating from the point $(-3, 0)$ is incident on the ellipse $16x^2 + 25y^2 = 400$ at the point P with ordinate 4. If the equation of the reflected ray after first reflection is $4x + 3y = \lambda$, then the value of 2^λ must be

4. If e be the eccentricity of the ellipse $4(x - 2y + 1)^2 + 9(2x + y + 2)^2 = 25$, then the value of $2187e^2$ must be

5. The orbit of the earth is an ellipse with eccentricity 1/60 with the sun at one focus the major axis being approximately 186×10^6 miles in length. If the shortest and longest distances of the earth from the sun are $\lambda \times 10^5$ miles and $\mu \times 10^5$ miles then the value of $(\lambda + \mu)$ must be

6. If the normals at the four points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then the value of $(x_1 + x_2 + x_3 + x_4) \times \left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right)$ must be

7. If two concentric ellipse be such that the *foci* of one be on the other and if 3/5 and 4/5 be their eccentricities. If θ be the angle between their axes, then the value of $1 + \sin\theta + \sin^4\theta$ must be

8. A variable point P on an ellipse of eccentricity 3/5 is joined to its *foci* S, S', then the locus of the incentre of the $\Delta PSS'$ is an ellipse. If λ be the eccentricity of the ellipse, then the value of $128\lambda^2$ must be

9. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at ends of latusrectum. If the area of an quadrilateral be λ sq unit, then the value of λ must be

Answers

Objective Questions Type I [Only one correct answer]

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (b)
21. (c)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (c)
31. (a)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (d)	**38.** (a)	**39.** (d)	**40.** (a)
41. (c)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (d)	**47.** (b)	**48.** (a)	**49.** (a)	**50.** (d)
51. (a)	**52.** (c)	**53.** (b)	**54.** (a)	**55.** (c)	**56.** (a)	**57.** (d)	**58.** (c)	**59.** (b)	**60.** (b)
61. (c)	**62.** (a)	**63.** (d)	**64.** (a)	**65.** (c)	**66.** (b)	**67.** (b)	**68.** (c)	**69.** (a)	**70.** (c)
71. (b)	**72.** (b)	**73.** (b)	**74.** (d)	**75.** (b)	**76.** (d)	**77.** (b)	**78.** (a)	**79.** (d)	**80.** (b)
81. (a)	**82.** (b)	**83.** (b)							

Objective Questions Type II [One or more than one correct answer(s)]

1. (c, d)	**2.** (a, d)	**3.** (a, c)	**4.** (a, b)	**5.** (a, d)
6. (a, c, d)	**7.** (a, b, d)	**8.** (a, b)	**9.** (a, c)	**10.** (a, d)

Linked-Comprehension Type

Passage 1 **1.** (b) **2.** (c) **3.** (a) **4.** (d) **5.** (d)

Passage 2 **1.** (b) **2.** (d) **3.** (a) **4.** (b) **5.** (c)

Passage 4 **1.** (c) **2.** (d) **3.** (b) **4.** (c) **5.** (b)

Passage 3 **1.** (c) **2.** (d) **3.** (b) **4.** (a) **5.** (d)

Numerical Grid-Based Problems

1.	0	4	1	0
2.	3	1	2	5
3.	4	0	9	6
4.	1	2	1	5
5.	1	8	6	0
6.	0	0	0	4
7.	0	0	0	3
8.	0	0	9	6
9.	0	0	2	7

Straight line JEE(Mains)

1. Let $P(-1,0)$ $Q(0,0)$ and $R(3,3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is- **[AIEEE 2007]**

(1) $\sqrt{3}x + y = 0$ (2) $x+\frac{\sqrt{3}}{2}y=0$ (3) $\frac{\sqrt{3}}{2}x+y=0$ (4) $x+\sqrt{3}y=0$

2. If one of the lines of $my^2 + (1-m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy=0$, then m is- **[AIEEE 2007]**

(1) $-\frac{1}{2}$ (2) -2 (3) 1 (4) 2

3. The perpendicular bisector of the line segment joining P (1, 4) and Q (k, 3) has y–intercept -4. Then a possible value of k is- **[AIEEE 2008]**

(1) 1 (2) 2 (3) -2 (4) -4

4. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are Perpendicular to a common line for : **[AIEEE 2009]**

(1) Exactly two values of p (2) More than two values of p

(3) No value of p (4) Exactly one value of p

5. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is : **[AIEEE-2010]**

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$ (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

6. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE 2011]**

Statement - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement - 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

7. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE 2011]**

(1) $(-1, 1]$ (2) $(0, \infty)$ (3) $[1, \infty)$ (4) $(-1, \infty)$

8. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE 2012]**

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) -4 (4) -2

9. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE 2012]**

(1) $\frac{11}{5}$ (2) $\frac{29}{5}$ (3) 5 (4) 6

10. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is : **[JEE-MAIN 2013]**

(1) $y = x + \sqrt{3}$ (2) $\sqrt{3}y = x - \sqrt{3}$ (3) $y = \sqrt{3}x - \sqrt{3}$ (4) $\sqrt{3}y = x - 1$

11. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1)(1, 1) and (1, 0) is : **[JEE-MAIN 2013]**

(1) $2 + \sqrt{2}$ (2) $2 - \sqrt{2}$ (3) $1 + \sqrt{2}$ (4) $1 - \sqrt{2}$

12. A light ray emerging from the point source placed at P(1, 3) is reflected at a point Q in the axis of *x*. If the reflected ray passes through the point R(6, 7), then the abscissa of Q is : **[JEE-MAIN Online 2013]**

(1) 3 (2) $\frac{7}{2}$ (3) 1 (4) $\frac{5}{2}$

13. If the three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ from a right – angled triangle then: **[JEE-MAIN Online 2013]**

(1) $a^2 - 6a - 12 = 0$ (2) $a^2 - 9a + 12 = 0$ (3) $a^2 - 9a + 18 = 0$ (4) $a^2 - 6a - 18 = 0$

14. If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is :- **[JEE-MAIN Online 2013]**

(1) –3 (2) –3/2 (3) –3/8 (4) –3/16

15. If the extremities of the base of an isoscelestriangle are the points (2a, 0) and (0, a) and the equation of one of the sides is$x = 2a$, then the area of the triangle, in square units, is : **[JEE-MAIN Online 2013]**

(1) $\frac{5}{2}a^2$ (2) $\frac{5}{4}a^2$ (3) $\frac{25a^2}{4}$ (4) $5a^2$

16. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$, andθ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers : **[JEE-MAIN Online 2013]**

Statement–1 : If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.

Statement–2 : $\theta_1 = \theta_2$ for all c_2 and c_3.

(1) Statement-1 is true and Statement - 2 is true, Statement-2 is not a correct explanation for Statement-1.

(2) Statement-1 is false and Statement-2 is true.

(3) Statement-1 is true and Statement-2 is false.

(4) Statement-1 is true and Statement - 2 is true, Statement-2 is a correct explanation for Statement-1.

17. Let A (-3, 2) and B (-2,1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x + 4y + 2 = 0$, then the vertex C lies on the line : **[JEE-MAIN Online 2013]**

(1) $4x + 3y + 5 = 0$ (2) $3x + 4y + 5 = 0$ (3) $3x + 4y + 3 = 0$ (4) $4x + 3y + 3 = 0$

18. If the image of point P(2, 3) in a line L is Q (4, 5) then, the image of point R (0, 0) in the same line is : **[JEE-MAIN Online 2013]**

(1) (4, 5) (2) (2, 2) (3) (3, 4) (4) (7, 7)

19. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then : **[JEE(Main)-2014]**

(1) $2bc - 3ad = 0$ (2) $2bc + 3ad = 0$ (3) $3bc - 2ad = 0$ (4) $3bc + 2ad = 0$

20. Let PS be the median of the triangle with vertices P (2, 2), Q (6, –1) and R (7, 3). The equation of the line passing through (1, –1) and parallel to PS is : **[JEE(Main)-2014]**

(1) $4x - 7y - 11 = 0$ (2) $2x + 9y + 7 = 0$ (3) $4x + 7y + 3 = 0$ (4) $2x - 9y - 11 = 0$

21. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a

(1) circle of radius $\sqrt{2}$ (2) circle of radius $\sqrt{3}$

(3) straight line parallel to x-axis (4) straight line parallel to y-axis

[JEE(Main)-2015]

22. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus ? **[JEE(Main)-2016]**

(1) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (2) (–3, –9) (3) (–3, –8) (4) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

23. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main)-2017]**

(1) $\left(2, \frac{1}{2}\right)$ (2) $\left(2, -\frac{1}{2}\right)$ (3) $\left(1, \frac{3}{4}\right)$ (4) $\left(1, -\frac{3}{4}\right)$

Straight line JEE (Advanced)

1. The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is

(A) 2 (B) 3 (C) 4 (D) 6

[JEE 2004 (Screening)]

2. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h,k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. [JEE 2005, Mains, 2]

3. (a) Let O(0, 0), P (3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

(A) $(4/3, 3)$ (B) $(3, 2/3)$ (C) $(3, 4/3)$ (D) $(4/3, 2/3)$

(b) Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

because

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. [JEE 2007, 3+3]

4. Consider the lines given by

$L_1 = x + 3y - 5 = 0$

$L_2 = 3x - ky - 1 = 0$

$L_3 = 5x + 2y - 12 = 0$

Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR.

	Column-I		**Column-II**
(A)	L_1, L_2, L_3 are concurrent, if	(P)	$k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q)	$k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(R)	$k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(S)	$k = 5$

[JEE 2008, 6]

5. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

6. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersect the x-axis, then the equation of L is **[JEE 2011, 3 (–1)]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

7. For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then **[JEE-Advanced 2013, 2]**

(A) a + b – c > 0

(B) a – b + c < 0

(C) a – b + c > 0

(D) a + b – c < 0

8. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines x – y = 0 and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

[JEE(Advanced)-2014, 3]

Answers:~

EXERCISE (JM)

1. 1 **2.** 3 **3.** 4 **4.** 4 **5.** 4 **6.** 1 **7.** 3 **8.** 4 **9.** 4 **10.** 2 **11.** 2 **12.** 4
13. 3 **14.** 4 **15.** 1 **16.** 4 **17.** 3 **18.** 4 **19.** 3 **20.** 2 **21.** 1 **22.** 4 **23.** 1

EXERCISE (JA)

1. A **2.** y = 2x + 1, y = – 2x + 1 **3.** (a) C; (b) C **4.** (A) S; (B) P,Q; (C) R; (D) P,Q,S
5. A **6.** B **7.** A or C or A,C **8.** 6

Circle JEE(Mains)

1. The point diametrically opposite to the point (1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is- **[AIEEE-2008]**

(1) (3, –4) (2) (–3, 4) (3) (–3, –4) (4) (3, 4)

2. Three distinct points A, B and C are given in the 2–dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point :- **[AIEEE-2009]**

(1) $\left(\frac{5}{2}, 0\right)$ (2) $\left(\frac{5}{3}, 0\right)$ (3) (0, 0) (4) $\left(\frac{5}{4}, 0\right)$

3. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for :-

(1) All except two values of p (2) Exactly one value of p **[AIEEE-2009]**

(3) All values of p (4) All except one value of p

4. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is :- **[AIEEE-2010]**

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

5. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if :- **[AIEEE-2010]**

(1) $-85 < m < -35$ (2) $-35 < m < 15$ (3) $15 < m < 65$ (4) $35 < m < 85$

6. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c > 0) touch each other if :- **[AIEEE-2011]**

(1) a = 2c (2) |a| = 2c (3) 2|a| = c (4) |a| = c

7. The equation of the circle passing through the points (1, 0) and (0, 1) and having the smallest radius is - **[AIEEE-2011]**

(1) $x^2 + y^2 + x + y - 2 = 0$ (2) $x^2 + y^2 - 2x - 2y + 1 = 0$

(3) $x^2 + y^2 - x - y = 0$ (4) $x^2 + y^2 + 2x + 2y - 7 = 0$

8. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : **[AIEEE-2012]**

(1) 5/3 (2) 10/3 (3) 3/5 (4) 6/5

9. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point : **[JEE (Main)-2013]**

(1) (–5, 2) (2) (2, –5) (3) (5, –2) (4) (–2, 5)

10. If a circle C passing through (4, 0) touches the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ externally at a point (1, –1), then the radius of the circle C is :- **[JEE-Main (on line)-2013]**

(1) $\sqrt{57}$ (2) $2\sqrt{5}$ (3) 4 (4) 5

11. If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x, then a equals :-

[JEE-Main (on line)-2013]

(1) ±4 (2) ±3 (3) 0 (4) ±2

12. Statement I : The only circle having radius $\sqrt{10}$ and a diameter along line $2x + y = 5$ is $x^2 + y^2 - 6x + 2y = 0$.

Statement II : $2x + y = 5$ is a normal to the circle $x^2 + y^2 - 6x + 2y = 0$. **[JEE-Main (on line)-2013]**

(1) Statement I is false, Statement II is true

(2) Statement I is true ; Statement II is false.

(3) Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.

(4) Statement I is true : Statement II is true ; Statement II is a correct explanation for Statement I.

13. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to :

[JEE(Main)-2014]

(1) $\frac{\sqrt{3}}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

14. The number of common tangents to the circle **[JEE(Main)-2015]**

$x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(1) 3 (2) 4 (3) 1 (4) 2

15. If one of the diameters of the circle, given by the euqation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is :- **[JEE(Main)-2016]**

(1) 10 (2) $5\sqrt{2}$ (3) $5\sqrt{3}$ (4) 5

16. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on :- **[JEE(Main)-2016]**

(1) A parabola (2) A circle

(3) An ellipse which is not a circle (4) A hyperbola

EXERCISE (JA)

1. Consider the two curves $C_1 : y^2 = 4x$; $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other **[JEE 2008, 3]**

2. Consider, $L_1 : 2x + 3y + p - 3 = 0$; $L_2 : 2x + 3y + p + 3 = 0$,

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

Statement-1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

Statement-2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True **[JEE 2008, 3]**

3. Comprehension (3 questions together):

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

(i) The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (B) $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x, y = 0$ **[JEE 2008, 4 + 4 + 4]**

4. Tangents drawn from the point P(l, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$ **[JEE 2009, 3]**

5. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is **[JEE 2009, 4]**

6. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is **[JEE 10, 3M]**

[**Note :** [k] denotes the largest integer less than or equal to k]

7. The circle passing through the point (–1,0) and touching the y-axis at (0,2) also passes through the point -

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4,0)

[JEE 2011, 3M, –1M]

8. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is **[JEE 2011, 4M]**

9. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is-
[JEE 2012, 3M, –1M]

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question 10 and 11

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

10. A common tangent of the two circles is **[JEE 2012, 3M, –1M]**

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = 4$ (D) $x + 2\sqrt{2}y = 6$

11. A possible equation of L is **[JEE 2012, 3M, –1M]**

(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

12. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ or y-axis is (are) **[JEE(Advanced) 2013, 3, (–1)]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

13. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then :- **[JEE(Advanced)-2014, 3]**

(1) radius of S is 8 (B) radius of S is 7

(3) centre of S is (–7, 1) (D) centre is S is (–8, 1)

14. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1,0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. then the locus of E passes through the point(s)- **[JEE(Advanced)-2016, 4(–2)]**

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

15. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ? **[JEE(Advanced)-2017, 3]**

Answers:~ CIRCLE

EXERCISE (JM)

1. 3	**2.** 4	**3.** 4	**4.** 3	**5.** 2	**6.** 4	**7.** 3	**8.** 2
9. 3	**10.** 4	**11.** 1	**12.** 1	**13.** 4	**14.** 1	**15.** 3	**16.** 1

EXERCISE (JA)

1. B	**2.** C	**3.** (i) D, (ii) A, (iii) D		**4.** B	**5.** 8	**6.** 3	**7.** D
8. 2	**9.** A	**10.** D	**11.** A	**12.** A,C	**13.** B,C	**14.** A,C	**15.** 2

Parabola JEE(Mains)

EXERCISE (JM)

1. A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at- **[AIEEE-2008]**

(1) (0, 2) (2) (1, 0) (3) (0, 1) (4) (2, 0)

2. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is :- **[AIEEE-2010]**

(1) $x = 1$ (2) $2x + 1 = 0$ (3) $x = -1$ (4) $2x - 1 = 0$

3. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$.

Statement–I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement–II : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$. **[JEE (Main)-2013]**

(1) Statement-I is true, Statement-II is true; statement-II is a **correct** explanation for Statement-I.

(2) Statement-I is true, Statement-II is true; statement-II is **not** a correct explanation for Statement-I.

(3) Statement-I is true, Statement-II is false.

(4) Statement-I is false, Statement-II is true.

4. Statement 1 : The slope of the tangent at any point P on a parabola, whose axis is the axis of x and vertex is at the origin, is inversely proportional to the ordinate of the point P.

Statement 2 : The system of parabolas $y^2 = 4ax$ satisfies a differential equation of degree 1 and order 1.

(1) Statement 1 is True Statement 2 is True, Statement 2 is a correct explanation for Statement 1.

(2) Statement 1 is True, Statement 2 is False.

(3) Statement 1 is True, Statement 2 is True statement 2 is not a correct explanation for statement 1.

(4) Statement 1 is False, Statement 2 is True **[JEE-Main (On line)-2013]**

5. **Statement 1 :** The line $x - 2y = 2$ meets the parabola, $y^2 + 2x = 0$ only at the point $(-2, -2)$

Statement 2 : The line $y = mx - \frac{1}{2m}$ $(m \neq 0)$ is tangent to the parabola, $y^2 = -2x$ at the point $\left(-\frac{1}{2m^2}, -\frac{1}{m}\right)$.

(1) Statement 1 is false; Statement 2 is true.

(2) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(3) Statement 1 is true; Statement 2 is false.

(4) Statement 1 is true; Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

[JEE-Main (On line)-2013]

6. The point of intersection of the normals tothe parabola $y^2 = 4x$ at the ends of its latus rectum is :

[JEE-Main (On line)-2013]

(1) (0, 3) (2) (2, 0) (3) (3, 0) (4) (0, 2)

7 The slope of the line touching both, the parabolas $y^2 = 4x$ and $x^2 = -32y$ is : **[JEE(Main)-2014]**

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{1}{8}$ (4) $\frac{2}{3}$

8. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :- **[JEE(Main)-2015]**

(1) $y^2 = 2x$ (2) $x^2 = 2y$ (3) $x^2 = y$ (4) $y^2 = x$

9. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the cente C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is : **[JEE(Main)-2016]**

(1) $x^2 + y^2 - 4x + 9y + 18 = 0$ (2) $x^2 + y^2 - 4x + 8y + 12 = 0$

(3) $x^2 + y^2 - x + 4y - 12 = 0$ (4) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

10. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :- **[JEE-Main 2017]**

(1) $4(\sqrt{2}+1)$ (2) $2(\sqrt{2}+1)$ (3) $2(\sqrt{2}-1)$ (4) $4(\sqrt{2}-1)$

EXERCISE (JA)

1. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$ **[JEE 2009, 4]**

(C) latus rectum is $\frac{2a}{3}$ (D) focus is $(a, 0)$

2. Let A and B be two distinct point on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be -

(A) $\frac{-1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $\frac{-2}{r}$ **[JEE 2010,3]**

3. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is **[JEE 2011,4]**

4. Let (x,y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0,0) to (x,y) in the ratio 1 : 3. Then the locus of P is- **[JEE 2011,3]**

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

5. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9,6), then L is given by - **[JEE 2011,4]**

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$ (C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

6. Let S be the focus of the parabola $y^2 = 8x$ & let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is **[JEE 2012, 4M]**

Paragraph for Question 7 and 8

Let PQ be a focal chord of the parabolas $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

7. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

[JEE(Advanced) 2013, 3, (–1)]

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$ (C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

8. Length of chord PQ is **[JEE(Advanced) 2013, 3, (–1)]**

(A) 7a (B) 5a (C) 2a (D) 3a

9. A line L : $y = mx + 3$ meets y – axis at E(0,3) and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0,y_0)$. The tangent to the parabola at $F(x_0,y_0)$ intersects the y-axis at $G(0,y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum. Match List-I with List-II and select the correct answer using the code given below the lists.

	List-I		**List-II**
P.	m =	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	y_0 =	3.	2
S.	y_1 =	4.	1

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	3	4	1	2
(C)	1	3	2	4
(D)	1	3	4	2

[JEE(Advanced) 2013, 3, (–1)]

10. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the point P, Q and the parabola at the points R,S. Then the area of the quadrilateral PQRS is -

(A) 3 (B) 6 (C) 9 (D) 15

[JEE(Advanced)-2014, 3(–1)]

Paragraph For Questions 11 and 12

Let a,r,s,t be nonzero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a,0).

11. The value of r is-

(A) $-\frac{1}{t}$ (B) $\frac{t^2+1}{t}$ (C) $\frac{1}{t}$ (D) $\frac{t^2-1}{t}$

[JEE(Advanced)-2014, 3(–1)]

12. If st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is-

(A) $\frac{(t^2+1)^2}{2t^3}$ (B) $\frac{a(t^2+1)^2}{2t^3}$ (C) $\frac{a(t^2+1)^2}{t^3}$ (D) $\frac{a(t^2+2)^2}{t^3}$

[JEE(Advanced)-2014, 3(–1)]

13. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is **[JEE 2015, 4M, –0M]**

14. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is **[JEE 2015, 4M, –0M]**

15. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is(are) the coordinates of P ?

[JEE 2015, 4M, –2M]

(A) $\left(4, 2\sqrt{2}\right)$ (B) $\left(9, 3\sqrt{2}\right)$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $\left(1, \sqrt{2}\right)$

16. The circle C_1: $x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then- **[JEE(Advanced)-2016, 4(–2)]**

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

17. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then-

(A) $SP = 2\sqrt{5}$

(B) $SQ : QP = \left(\sqrt{5}+1\right) : 2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

[JEE(Advanced)-2016, 4(–2)]

18. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

[JEE(Advanced)-2017, 4(–2)]

(A) $p = 5, h = 4, k = -3$

(B) $p = -1, h = 1, k = -3$

(C) $p = -2, h = 2, k = -4$

(D) $p = 2, h = 3, k = -4$

EXERCISE (JM)

1. 2 **2.** 3 **3.** 2 **4.** 3 **5.** 4 **6.** 3 **7.** 1 **8.** 2 **9.** 2 **10.** (Bonus) or 4

EXERCISE (JA)

1. A,D **2.** C,D **3.** 2 **4.** C **5.** A,B,D **6.** 4 **7.** D **8.** B **9.** A
10. D **11.** D **12.** B **13.** 2 **14.** 4 **15.** A,D **16.** A,B,C **17.** A,C,D **18.** D

EXERCISE (JM)

1. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is 1/2. Then the length of the semi-major axis is- **[AIEEE-2008]**

(1) 8/3 (2) 2/3 (3) 4/3 (4) 5/3

2. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is :- **[AIEEE-2009]**

(1) $4x^2 + 48y^2 = 48$ (2) $4x^2 + 64y^2 = 48$ (3) $x^2 + 16y^2 = 16$ (4) $x^2 + 12y^2 = 16$

3. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{2/5}$ is :- **[AIEEE-2011]**

(1) $3x^2 + 5y^2 - 15 = 0$ (2) $5x^2 + 3y^2 - 32 = 0$

(3) $3x^2 + 5y^2 - 32 = 0$ (4) $5x^2 + 3y^2 - 48 = 0$

4. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[AIEEE-2012]**

(1) $x^2 + 4y^2 = 16$ (2) $4x^2 + y^2 = 4$ (3) $x^2 + 4y^2 = 8$ (4) $4x^2 + y^2 = 8$

5. **Statement–1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement–2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. **[AIEEE-2012]**

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

6. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at (0, 3) is : **[JEE (Main)-2013]**

(1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$

(3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

7. If a and c are positive real number and the ellipse $\frac{x^2}{4c^2}+\frac{y^2}{c^2}=1$ has four distinct points in common with the circle $x^2 + y^2 = 9a^2$, then **[JEE-Main (On line)-2013]**

(1) $6ac + 9a^2 - 2c^2 > 0$ (2) $6ac + 9a^2 - 2c^2 < 0$

(3) $9ac - 9a^2 - 2c^2 < 0$ (4) $9ac - 9a^2 - 2c^2 > 0$

8. Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is - **[JEE-Main (On line)-2013]**

(1) $y + 3 = 0$ (2) $3y + 1 = 0$ (3) $3y - 1 = 0$ (4) $y - 3 = 0$

9. Let the equations of two ellipses be $E_1 : \frac{x^2}{3}+\frac{y^2}{2}=1$ and $E_2 : \frac{x^2}{16}+\frac{y^2}{b^2}=1$. If the product of their eccentricities is $\frac{1}{2}$, then the length of the minor axis of ellipse E_2 is :- **[JEE-Main (On line)-2013]**

(1) 9 (2) 8 (3) 2 (4) 4

10. If the curves $\frac{x^2}{\alpha}+\frac{y^2}{4}=1$ and $y^3 = 16x$ intersect at right angles, then a value of α is : **[JEE-Main (On line)-2013]**

(1) $\frac{4}{3}$ (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) 2

11. A point on the ellipse, $4x^2 + 9y^2 = 36$, where the normal is parallel to the line, $4x - 2y - 5 = 0$, is : **[JEE-Main (On line)-2013]**

(1) $\left(\frac{8}{5},-\frac{9}{5}\right)$ (2) $\left(-\frac{9}{5},\frac{8}{5}\right)$ (3) $\left(\frac{8}{5},\frac{9}{5}\right)$ (4) $\left(\frac{9}{5},\frac{8}{5}\right)$

12. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is : **[JEE(Main)-2014]**

(1) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ (2) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

(3) $(x^2 + y^2)^2 = 6x^2 + 2y^2$ (4) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

13. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is : **[JEE(Main)-2015]**

(1) $\frac{27}{2}$ (2) 27 (3) $\frac{27}{4}$ (4) 18

14. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directices is $x = -4$, then the equation of the normal to it at $\left(1,\frac{3}{2}\right)$ is :- **[JEE-Main 2017]**

(1) $x + 2y = 4$ (2) $2y - x = 2$ (3) $4x - 2y = 1$ (4) $4x + 2y = 7$

EXERCISE (JA)

1. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are **[JEE 2008, 4]**

   (A) $x^2 + 2\sqrt{3}\,y = 3 + \sqrt{3}$
   (B) $x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$
   (C) $x^2 + 2\sqrt{3}\,y = 3 - \sqrt{3}$
   (D) $x^2 - 2\sqrt{3}\,y = 3 - \sqrt{3}$

2. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[JEE 2009, 3]**

   (A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

3. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the point **[JEE 2009, 3]**

   (A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

4. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\frac{A}{2}$.

   If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then **[JEE 2009, 4]**

   (A) $b + c = 4a$
   (B) $b + c = 2a$
   (C) locus of point A is an ellipse.
   (D) locus of point A is a pair of straight lines.

Comprehension: 7 to 9

Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B. **[JEE 2010, 3+3+3]**

5. The coordinates of A and B are

   (A) (3,0) and (0,2)
   (B) $\left(-\frac{8}{5}, \frac{2\sqrt{261}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$
   (C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0,2)
   (D) (3,0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

6. The orthocenter of the triangle PAB is

   (A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

7. The equation of the locus of the point whose distances from the point P and the line AB are equal, is -

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

8. The ellipse $E_1 : \frac{x^2}{9} + \frac{x^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is - **[JEE 2012, 3M, –1M]**

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

9. A vertical line passing through the point (h,0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE-Advanced 2013, 4, (–1)]**

10. **List-I** **List-II**

P. Let $y(x) = \cos(3\cos^{-1} x)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2 - 1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals — **1.** 1

Q. Let $A_1, A_2, \ldots\ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, k = 1, 2, ... n. If $\left|\sum_{k=1}^{n-1}(\overrightarrow{a_k} \times \overrightarrow{a_{k+1}})\right| = \left|\sum_{k=1}^{n-1}(\overrightarrow{a_k} . \overrightarrow{a_{k+1}})\right|$, then the minimum value of n is — **2.** 2

R. If the normal from the point P(h, 1) on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line x + y = 8, then the value of h is — **3.** 8

S. Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is — **4.** 9

Codes :

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

[JEE(Advanced)-2014, 3(–1)]

11. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1,0)$ and $(f_2,0)$ where $f_1>0$ and $f_2<0$. Let P_1 and P_2 be two parabolas with a common vertex at (0,0) and with foci at $(f_1,0)$ and $(2f_2,0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2,0)$ and T_2 be a tangent to P_2 which passes through $(f_1,0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is

[JEE 2015, 4M, –0M]

12. Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2+(y-1)^2=2$. The straight line $x+y=3$ touches the curves S, E_1 and E_2 at P,Q and R, respectively. Suppose that $PQ=PR=\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is(are)

[JEE 2015, 4M, –0M]

(A) $e_1^2+e_2^2=\frac{43}{40}$ (B) $e_1e_2=\frac{\sqrt{7}}{2\sqrt{10}}$ (C) $\left|e_1^2-e_2^2\right|=\frac{5}{8}$ (D) $e_1e_2=\frac{\sqrt{3}}{4}$

PARAGRAPH :

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1<0$ and $x_2>0$, be the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

13. The orthocentre of the triangle F_1MN is- **[JEE(Advanced)-2016, 4(–2)]**

(A) $\left(-\frac{9}{10},0\right)$ (B) $\left(\frac{2}{3},0\right)$ (C) $\left(\frac{9}{10},0\right)$ (D) $\left(\frac{2}{3},\sqrt{6}\right)$

14. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is-

[JEE(Advanced)-2016, 3(0)]

(A) 3 : 4 (B) 4 : 5 (C) 5 : 8 (D) 2 : 3

EXERCISE (JM)

1. 1 **2.** 4 **3.** 3 **4.** 1 **5.** 3 **6.** 1 **7.** 4 **8.** 3 **9.** 4
10. 1 **11.** 4 **12.** 3 **13.** 2 **14.** 3

EXERCISE (JA)

1. B,C **2.** D **3.** C **4.** B, C **5.** D **6.** C **7.** A **8.** C
9. 9 **10.** A **11.** 4 **12.** A,B **13.** A **14.** C

Hyperbola JEE(Mains)

1. The equation of the hyperbola whose foci are (–2,0) and (2, 0) and eccentricity is 2 is given by : **[AIEEE-2011]**

(1) $-3x^2 + y^2 = 3$ (2) $x^2 - 3y^2 = 3$ (3) $3x^2 - y^2 = 3$ (4) $-x^2 + 3y^2 = 3$

2. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on : **[JEE-Main (On line)-2013]**

(1) $\frac{2}{x^2} - \frac{4}{y^2} = 1$ (2) $\frac{4}{x^2} - \frac{2}{y^2} = 1$ (3) $\frac{4}{x^2} + \frac{2}{y^2} = 1$ (4) $\frac{2}{x^2} + \frac{4}{y^2} = 1$

3. A common tangent to the conics $x^2 = 6y$ and $2x^2 - 4y^2 = 9$ is : **[JEE-Main (On line)-2013]**

(1) $x + y = \frac{9}{2}$ (2) $x + y = 1$ (3) $x - y = \frac{3}{2}$ (4) $x - y = 1$

4. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[JEE-Main 2016]**

(1) $\sqrt{3}$ (2) $\frac{4}{3}$ (3) $\frac{4}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

5. A hyperbola passes through the point $P(\sqrt{2},\sqrt{3})$ and has foci at (± 2, 0). Then the tangent to this hyperbola at P also passes through the point : **[JEE-Main 2017]**

(1) $(-\sqrt{2},-\sqrt{3})$ (2) $(3\sqrt{2},2\sqrt{3})$ (3) $(2\sqrt{2},3\sqrt{3})$ (4) $(\sqrt{3},\sqrt{2})$

Hyperbola JEE(Advanced)

1. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents **[JEE 2008, 3]**

(A) four straight lines, when $c = 0$ and a, b are of the same sign.

(B) two straight lines and a circle, when $a = b$, and c is of sign opposite to that of a.

(C) two straight lines and a hyperbola, when a & b are of the same sign and c is of sign opposite to that of a.

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

2. Consider a branch of the hyperbola, $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is **[JEE 2008, 3]**

(A) $1-\sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}}-1$ (C) $1+\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}}+1$

3. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is **[JEE 2009, 3]**

(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

4. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then **[JEE 2009, 4]**

(A) Equation of ellipse is $x^2 + 2y^2 = 2$ (B) The foci of ellipse are $(\pm 1, 0)$

(C) Equation of ellipse is $x^2 + 2y^2 = 4$ (D) The foci of ellipse are $(\pm\sqrt{2}, 0)$

5. Match the conics in **Column I** with the statements/expressions in **Column II.** **[JEE 2009, 8]**

	Column I		**Column II**
(A)	Circle	(p)	The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B)	Parabola	(q)	Points z in the complex plane satisfying $\|z+2\| - \|z-2\| = \pm 3$
(C)	Ellipse	(r)	Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$
(D)	Hyperbola		
		(s)	The eccentricity of the conic lies in the interval $1 \le x < \infty$
		(t)	Points z in the complex plane satisfying $Re(z+1)^2 = \|z\|^2 + 1$

Comprehension: 6 & 7 **[JEE 2010, 3+3]**

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

6. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is -

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$ (C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

7. Equation of the circle with AB as its diameter is -

(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

8. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

[JEE 2010, 3]

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then - **[JEE 2011, 4]**

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(B) a focus of the hyperbola is (2,0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2-3y^2=3$

10. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at (9,0), then the eccentricity of the hyperbola is - **[JEE 2011, 3]**

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

11. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line 2x – y = 1. The points of contact of the tangents on the hyperbola are **[JEE 2012, 4M]**

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3},-2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3},2\sqrt{2}\right)$

12. Consider the hyperbola H : $x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M . If (l, m) is the centroid of the triangle ΔPMN, then the correct expression(s) is(are) **[JEE 2015, 4M, –0M]**

(A) $\frac{dl}{dx_1}=1-\frac{1}{3x_1^2}$ for $x_1 > 1$ (B) $\frac{dm}{dx_1}=\frac{x_1}{3\left(\sqrt{x_1^2-1}\right)}$ for $x_1 > 1$

(C) $\frac{dl}{dx_1}=1+\frac{1}{3x_1^2}$ for $x_1 > 1$ (D) $\frac{dm}{dy_1}=\frac{1}{3}$ for $y_1 > 0$

13. If 2x – y + 1 = 0 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{16}=1$, then which of the following CANNOT be sides of a right angled triangle ? **[JEE(Advanced)-2017, 4(–2)]**

(A) 2a, 4, 1 (B) 2a, 8, 1 (C) a, 4, 1 (D) a, 4, 2

Column 1, 2 and 3 contain conics, equation of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

14. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only **CORRECT** combination ?

[JEE(Advanced)-2017, 3(–1)]

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

15. If a tangent to a suitable conic (Column 1) is found to be y = x + 8 and its point of contact is (8,16), then which of the following options is the only **CORRECT** combination ?

[JEE(Advanced)-2017, 3(–1)]

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

16. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1,1), then which of the following options is the only **CORRECT** combination for obtaining its equation ?

[JEE(Advanced)-2017, 3(–1)]

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

EXERCISE (JM)

1. 3 **2.** 2 **3.** 3 **4.** 4 **5.** 3

EXERCISE (JA)

1. B **2.** B **3.** D **4.** A, B **5.** (A) p, (B) s,t; (C) r ; (D) q,s **6.** B
7. A **8.** 2 **9.** B,D **10.** B **11.** A,B **12.** A,B,D **13.** B,C,D
14. D **15.** A **16.** D